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THE

MATHEMATICAL MONTHLY.

SEPTEMBER, 1859.

EDITED BY

J. D. RUNKLE, A.M., A.A.S.

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NOTICE.

THIS Number closes the First Volume of the MATHEMATICAL MONTHLY. The next Number will be promptly issued. It is desirable that its circulation should be largely increased, which can be readily done by a little effort on the part of present Subscribers. The MONTHLY may then be enlarged, and more space be devoted to such matters as will make it more valuable to Teachers and Students. The Prizes will be continued, with some modifications; and it is hoped that those for whom they are intended, will insure their continuance, by making an effort to increase the subscription list. All additional subscriptions should be sent to the editor immediately.

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THE

MATHEMATICAL MONTHLY.

Vol. I....SEPTEMBER, 1859....No. XII.

REPORT OF THE JUDGES UPON THE SOLUTIONS OF THE PRIZE PROBLEMS IN No. VIII., Vol. I.

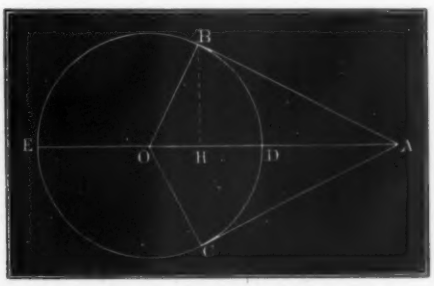
The first Prize is awarded to GEORGE A. OSBORNE, Jr., of the Lawrence Scientific School, Cambridge, Mass.

The second Prize is awarded to GEORGE W. JONES, Jr., Senior, Yale College, New Haven, Ct.

PRIZE SOLUTION OF PROBLEM I.

"If x be the distance of the eye from the centre of a sphere, of which the radius is r, prove that the visible part of its surface is to the invisible as $x-r : x+r$."

Let A denote the position of the eye, O the centre of the sphere, and $BDCE$ a great circle of the sphere cut by a plane passing through A and O. Draw the tangents AB and AC, the radii OB and OC, and BH perpendicular to AE. Then



$$\frac{AO}{OB} = \frac{OB}{OH} \text{ or, } \frac{x}{r} = \frac{r}{OH}.$$

$$\therefore \frac{x-r}{x+r} = \frac{r-OH}{r+OH} = \frac{DH}{HE}.$$

Since zones of the same sphere are to each other as their alti-

tudes, the visible and invisible portions or zones are to each other as their altitudes $DH : HE :: x - r : x + r$.

This solution is by GEORGE A. OSBORNE, JR.

PRIZE SOLUTION OF PROBLEM II.

"Transform the series

$$1 + 8 + 19 + 34 + 53 + 76 + \&c.,$$

so as to find the sum of n terms by means of the usual formula for summing the squares of the natural numbers."

Assume

$$A + Bn + Cn^2 + Dn^3 = 1 + 8 + 19 + 34 + 53 + 76 + \ldots.$$

and make $n = 1, 2, 3, 4$, successively. We get

$$\begin{aligned} &A + B + C + D = 1, \\ &A + 2B + 4C + 8D = 1 + 8, \\ &A + 3B + 9C + 27D = 1 + 8 + 19, \\ &A + 4B + 16C + 64D = 1 + 8 + 19 + 34. \end{aligned}$$

From these equations we get

$$D = \tfrac{2}{3},\ C = \tfrac{3}{2},\ B = -\tfrac{7}{6},\ A = 0;$$

and, by substitution, the assumed formula becomes

$$\begin{aligned} \frac{(4n^2 + 9n - 7)n}{6} &= \frac{2(2n^2 + 3n + 1)n}{6} + \frac{n(3n - 9)}{6} \\ &= \frac{n(n+1)(2n+1)}{3} + \frac{(n-3)n}{2}, \end{aligned}$$

as the sum of the series.

$$\therefore 1 + 8 + 19 + \ldots. 2n^2 + n - 2 = 2(1 + 4 + 9 + \ldots n^2) + \frac{n(n-3)}{2}.$$

The part $\frac{n(n-3)}{2}$ is the sum of an arithmetical series, of which the first term is 2, the last term $n - 2$, the number of terms $n - 3$, and the common difference 1. This is also shown by

$$\begin{aligned} 1 + 8 + 19 + 34 + \ldots. &= 1 + 2.4 + (1 + 2.9) \\ &\quad + (2 + 2.16) + (3 + 2.25) + \ldots. \end{aligned}$$

$$= 2(1 + 2^2 + 3^2 + 4^2 + \ldots . n^2) + (2 + 3 + 4 + \ldots . n - 2)$$
$$= n\frac{(n+1)(2n+1)}{3} + \frac{(n-3)n}{2}.$$

This solution is by GUSTAVUS FRANKENSTEIN.

PRIZE SOLUTION OF PROBLEM III.

"If a, b, c are the sides of a spherical triangle, and A, B, C the opposite angles, prove that

$$\sin b \sin c + \cos b \cos c \cos A = \sin B \sin C - \cos B \cos C \cos a."$$

If we multiply the elementary formulas of spherical trigonometry,

$$\cos a = \sin b \sin c \cos A + \cos b \cos c$$
$$\cos A = \sin B \sin C \cos a - \cos B \cos C,$$

by $\cos A$ and $\cos a$ respectively, we have

$$(1) \qquad \begin{aligned} \cos a \cos A &= \sin b \sin c \cos^2 A + \cos b \cos c \cos A \\ &= \sin B \sin C \cos^2 a - \cos B \cos C \cos a. \end{aligned}$$

Also since $$\frac{\sin a}{\sin A} = \frac{\sin b}{\sin B} = \frac{\sin c}{\sin C},$$

we have $$\frac{\sin b \sin c}{\sin B \sin C} = \frac{\sin^2 a}{\sin^2 A} = \frac{1 - \cos^2 a}{1 - \cos^2 A}.$$

$$(2) \quad \therefore \sin b \sin c (1 - \cos^2 A) = \sin B \sin C (1 - \cos^2 a),$$

Adding (2) to second and third members of (1) gives

$$\sin b \sin c + \cos b \cos c \cos A = \sin B \sin C - \cos B \cos C \cos a.$$

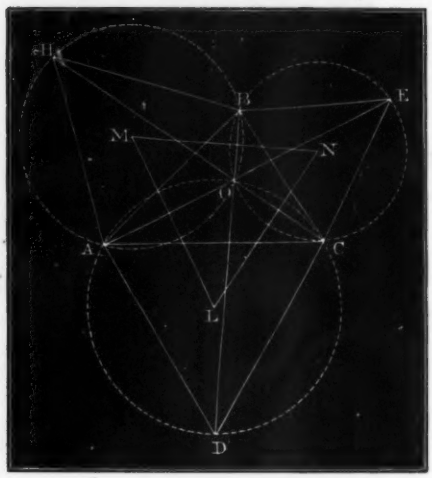
This solution is by GEORGE A. OSBORNE, Jr.

PRIZE SOLUTION OF PROBLEM IV.

"If, on the sides of a given plane triangle, equilateral triangles be constructed, prove that the triangle formed by joining the centres of these equilateral triangles will also be equilateral; also prove that the straight lines joining the vertices of the equilateral triangles and the opposite angles of the given triangle are equal, and all intersect in the same point."

Let $A\,B\,C$ be the given triangle. Construct the equilateral triangles $A\,C\,D$, $A\,H\,B$, and $B\,E\,C$, and circumscribe a circle about

each. These circles will intersect at the same point, O; for consider O to be the intersection of the two circles AHB and BEC, and draw OA, OB, and OC. Since the angles AHB and BEC are each $= 60°$, their supplements AOB and BOC are each $= 120°$. $\therefore$ the angle AOC is also $= 120°$, and is hence the supplement of ADC; therefore the circle ADC must pass through the point O. Moreover, the lines joining the centres of the equilateral triangles, or of their circumscribing circles, being perpendicular to the common chords OA, OB, and OC, will meet at angles of 60° and form an equilateral triangle.



Draw OE, OD, and OH, then the angle $EOC = EBC$, which is the supplement of COA; therefore OA and OE form one right line. Similarly it may be shown that HOC and BOD are right lines, which, together with AOE, intersect in a common point O.

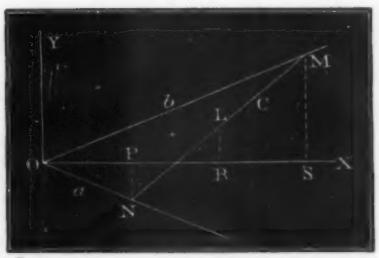
Since the two angles BCD and ECA are equal and included between equal sides, the triangles BCD and ECA are equal and $AE = BD$; similarly it may be shown that $BD = HC$.

This solution is by George A. Osborne, Jr.

Prize Solution of Problem V.

"If an angle (A), and the sum of the squares of the sides of a plane triangle, be given ($= 8a^2$), prove that the curve which continually bisects the side opposite to A is an ellipse, and determine the numerical values of its principal diameters when $A = 60°$, and $a = 10$."

Let MON be the given angle A, and MN one of the positions of the opposite side. Denote by b, c, and d the sides OM, MN, and NO respectively. Draw OX bisecting the angle A, and to this, as the coördinate axis of X and the vertex O as origin, let the required locus be referred. L, the middle point of MN, is a point of this locus. Draw the perpendiculars NP, LR, and MS; then $OR=x$, and $LR=y$,



$$OR=OP+\tfrac{1}{2}(OS-OP)=\tfrac{1}{2}(OS+OP), \text{or} x=\tfrac{1}{2}(b+d)\cos\tfrac{A}{2}; \quad (1)$$

$$LR=MS-\tfrac{1}{2}(MS+PN)=\tfrac{1}{2}(MS-PN), \text{or} y=\tfrac{1}{2}(b-d)\sin\tfrac{A}{2}; \quad (2)$$

squaring (1) and (2) gives

$$\frac{x^2}{\cos^2\frac{A}{2}}=\tfrac{1}{4}(b+d)^2 \ (3), \text{ and } \frac{y^2}{\sin^2\frac{A}{2}}=\tfrac{1}{4}(b-d)^2 \ldots (4);$$

multiplying (3) and (4) by $(2-\cos A)$ and $(2+\cos A)$ respectively and adding, gives

$$x^2\frac{2-\cos A}{\cos^2\frac{A}{2}}+y^2\frac{2+\cos A}{\sin^2\frac{A}{2}}=\tfrac{1}{4}(b+d)^2(2-\cos A)+\tfrac{1}{4}(b-d)^2(2+\cos A)$$
$$=b^2+d^2-b\,d\cos A.$$

But by the conditions of the problem $b^2+c^2+d^2=8a^2$, also we have $c^2=b^2+d^2-2\,b\,d\cos A$; $\therefore\ b^2+d^2-b\,d\cos A=4a^2$, by eliminating c^2.

Hence
$$x^2\frac{2-\cos A}{\cos^2\frac{A}{2}}+y^2\frac{2+\cos A}{\sin^2\frac{A}{2}}=4a^2, \text{or}$$

$$x^2\frac{2-\cos A}{1+\cos A}+y^2\frac{2+\cos A}{1-\cos A}=2a^2,$$

which is the equation of an ellipse referred to its axes.

If $A=60°$, $\cos A=\frac{1}{2}$, and $a=10$, the equation of the ellipse reduces to $x^2+5y^2=200$; if in this, $y=0$, $x=10\sqrt{2}$, if $x=0$, $y=2\sqrt{10}$; hence the values of the semi-axes are $10\sqrt{2}$ and $2\sqrt{10}$.

If $A = 90°$, we have $x^2 + y^2 = a^2$; hence if the given angle is a right angle, the locus is a circle whose centre is at the vertex of the right angle and radius $= a$.

This solution is by GEORGE A. OSBORNE, Jr.

JOSEPH WINLOCK.
CHAUNCEY WRIGHT.
TRUMAN HENRY SAFFORD.

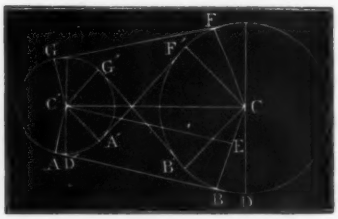
SOLUTION OF PRIZE PROBLEM V., No. VII.

By J. B. HENCK, C. E., Boston.

THE problem of the belt and pulleys admits, perhaps, of a simpler solution, if we develop the arc and cosine in terms of the sine, instead of developing the sine and cosine in terms of the arc, as is done in the second solution, page 360. Another advantage is, that in this way a quite accurate approximate formula may be obtained, especially in the case of the open belt.

Let $B\,C = R$, $A\,C' = r$, $C\,C' = a$, and the length of the belt $= l$.

I. When the belt is open, denote the angle $A\,C'\,D' = B\,C\,D = C\,C'\,E$ by φ. Then the straight portions of the belt $= 2\,A\,B = 2\,C'\,E = 2a\cos\varphi$; also $R - r = CE = a\sin\varphi$. The length of the arc $B\,D\,F = R(\pi + 2\varphi) = R\pi + 2R\varphi$; that of the arc $A\,G = r(\pi - 2\varphi) = (R - a\sin\varphi)(\pi - 2\varphi) = R\pi - 2R\varphi - a\pi\sin\varphi + 2a\varphi\sin\varphi$. Adding the straight and curved portions, we have



$$2a\cos\varphi + 2R\pi - a\pi\sin\varphi + 2a\varphi\sin\varphi = l;$$

$$(1) \qquad \therefore\ 2\cos\varphi - \pi\sin\varphi + 2\varphi\sin\varphi = \frac{l - 2R\pi}{a}.$$

Let $\sin\varphi = x$. Then $\varphi = x + \frac{1}{6}x^3 + \frac{3}{40}x^5 +$ &c., and $\cos\varphi = \sqrt{1-\sin^2\varphi}$ $= \sqrt{1-x^2} = 1 - \frac{1}{2}x^2 - \frac{1}{8}x^4 - \frac{1}{16}x^6 -$ &c. Substituting these values of φ and $\cos\varphi$ in (1), we have

$$2\left(1 - \tfrac{1}{2}x^2 - \tfrac{1}{8}x^4 - \tfrac{1}{16}x^6 - \text{\&c.}\right) - \pi x + 2x\left(x + \tfrac{1}{6}x^3 + \tfrac{3}{40}x^5 + \text{\&c.}\right) = \frac{l - 2R\pi}{a}.$$

$$(2) \qquad \therefore x^2 - \pi x + \tfrac{1}{12}x^4 + \tfrac{1}{40}x^6 + \text{\&c.} = \frac{l - 2R\pi}{a} - 2.$$

The value of x being found from (2), we have $r = R - a\sin\varphi$ $= R - ax$.

Now since x is a proper fraction, generally small, we may obtain an approximate formula by neglecting $\frac{1}{12}x^4 + \frac{1}{40}x^6 +$ &c.

Hence,

$$x^2 - \pi x = \frac{l - 2R\pi}{a} - 2;$$

$$\therefore x = \tfrac{1}{2}\pi \pm \sqrt{\tfrac{1}{4}\pi^2 + \frac{l - 2R\pi}{a} - 2};$$

$$(3) \qquad \therefore x = 1.5708 \pm \sqrt{.46740110 + \frac{l - 6.28318531R}{a}}.$$

II. When the belt is crossed, denote the angle $A'C'D' = B'CD$ $= 90° - B'CC'$ by φ. Then if a perpendicular be dropped from C' on CB' produced, it will be seen that each of the straight parts of the belt $= A'F' = B'G' = CC'\cos\varphi = a\cos\varphi$; also that $R + r = a\sin\varphi$. The length of the arc $B'DF' = R(\pi + 2\varphi)$; that of the arc $A'D'G'$ $= r(\pi + 2\varphi)$; so that the sum of the curved portions $= (R + r)$ $(\pi + 2\varphi) = a\sin\varphi(\pi + 2\varphi)$.

$$\therefore 2a\cos\varphi + a\pi\sin\varphi + 2a\varphi\sin\varphi = l;$$

$$\therefore 2\cos\varphi + \pi\sin\varphi + 2\varphi\sin\varphi = \frac{l}{a}.$$

Substituting the same values of φ and $\cos\varphi$ as above, we have

$$(4) \qquad x^2 + \pi x + \tfrac{1}{12}x^4 + \tfrac{1}{40}x^6 + \text{\&c.} = \frac{l}{a} - 2.$$

The value of x being found from (4), we have $r = a \sin \varphi - R = a x - R$.

The approximate formula is

$$x^2 + \pi x = \frac{l}{a} - 2;$$

$$\therefore x = -\tfrac{1}{2}\pi \pm \sqrt{\tfrac{1}{4}^2 + \frac{l}{a} - 2};$$

$$\therefore x = -1.5708 \pm \sqrt{.46740110 + \frac{l}{a}}. \tag{5}$$

The formula in this case is not so accurate as in the first case, because x is generally a larger fraction. In both cases a second approximation is very readily made by subtracting from the quantity under the radical in (3) and (5) the value of as many terms of $\frac{1}{12}x^4 + \frac{1}{40}x^6 +$ &c., as may be thought necessary, computed from the first approximate value of x. A third approximation may be made in a similar way.

This method of approximation, it is evident, may be applied to finding the incommensurable roots of numerical equations of any degree; since it is easy to transform such an equation so that its roots shall be proper fractions. The method occurred to me several years ago, while solving this same problem. It is more than probable, however, that so obvious a process has been tried before, and found more laborious than the ordinary ones.

NOTES AND QUERIES.

[Continued from page 264.]

8. The following tabular summary of the notation and definitions of algebra may be used with advantage in teaching beginners. Write the table on the blackboard, or on a large card, and let the class review it daily until it is fixed in the mind. Let a, b, c, &c.

denote any numbers whatever; their relations and the corresponding symbols have the following names: —

Symbol.	Name of Symbol.	Relation.	Name of Relation.	
$+$	*Plus.*	$a+b$	*Sum*:	b added to a.
$-$	*Minus.*	$a-b$	*Diff.*:	b subtracted from a.
$\times$, ., nothing.	*Multiplied by.*	$a\times b, a.b, ab$	*Product*:	a multiplied by b.
$\div$ fraction.	*Divided by.*	$a\div b, \frac{a}{b}$	*Quotient*:	a divided by b.
$=$	*Equals.*	$a+b=c-d$	*Equation*:	sum equals diff.
$<$	*Less than.*	$a<b$	*Inequality*:	a less than b.
$>$	*Greater than*	$a>b$	*Inequality*:	a greater than b.
:	*Ratio.*	$a:b$	*Quotient*:	a divided by b.
::	*Equality of ratios.*	$a:b::c:d$	*Proportion*:	equality of ratios.
——	*Vinculum or bar.*	$\overline{a+b}\times c, \frac{a+b}{c}$	Sum multiplied or divided by c.	
( )	*Parenthesis.*	$a-(b-c+d)$	$(b-c+d)$ as a whole, subtr'd from a.	
$(\ )^n$	*Exponent.*	$(a+b)^n, c^n$	*Power*: $a+b$ or c, taken n times as a factor.	
$\sqrt[n]{\ }$, $(\ )^{\frac{1}{n}}$	*Root-sign.*	$\sqrt[n]{a+b}, (a+b)^{\frac{1}{n}}$	*Root*: nth root of $a+b$.	
$'$ $''$ $'''$ &c.	*Accents.*	a', a'', a''' &c.	Symmetry of notation.	

This idea each teacher can extend and modify at pleasure.

Remark 1. These symbols, or many of them, are usually defined in the arithmetics, but hardly ever used. Why is this? How easy it would be to make pupils entirely familiar with them in connection with *numbers*, about which their ideas are clear and fixed. All these symbols ought constantly to be used with small numbers, until they are indelibly associated in the mind with the processes they indicate; and all operations ought to be indicated before they are performed. Thus,

$$2\sqrt{\frac{(2+5-3)\times 8+12}{11}} = 2\sqrt{\frac{4\times 8+12}{11}} = 2\sqrt{\tfrac{44}{11}} = 2\sqrt{4} = 4.$$

If this were done, how easy and obvious the transition to algebra would be. Arithmetic, and the simple elements of algebra, are very intimately connected; and by thus gradually passing from one to the other, we shall make the notation, which has been too much

considered peculiar to algebra, useful in explaining the operations of arithmetic; while, on the other hand, these arithmetical operations furnish the simplest practical illustrations of the results of algebra.

After the pupil is once familiar with all the symbols, how easy it would be to introduce into numerical problems the answer as a quantity, not perhaps at first denoting it by a letter, but by some significant word.

Remark 2. It is usually much better to write a proportion in the form $\frac{a}{b}=\frac{c}{d}$, than $a:b::c:d$. Because, in all the transformations,

$$\frac{a}{c}=\frac{b}{d}\,,\ \frac{d}{b}=\frac{c}{a}\,,\ \frac{a}{a+b}=\frac{c}{c+d}\,,\ \frac{a}{c}=\frac{a+b}{c+d}\,,\ \frac{a-b}{a+b}=\frac{c-d}{c+d}\,,\ \text{\&c.,}$$

the symmetry, which is more directly apparent, is of great service.

9. *Simplification of the Expression* $\sqrt{a\pm c\sqrt{d}}$. The remarks of Mr. JOHN M. RICHARDSON, in No. II., on the reduction of expressions of this form, have induced me to refer the readers of the MONTHLY to BENEDICT'S Algebra, page 152, where the subject is treated in the same manner. We will add a single example, which may be taken as the representative of others, that at first seems to be incapable of reduction in the manner proposed. Simplify the expression $\sqrt{2x\pm 2\sqrt{x^2-1}}$. Here the correspondence between the rational and radical parts of the expression may be seen as follows.

The rational part $2x=(x+1)+(x-1)$.

The radical part $2\sqrt{x^2-1}=2\sqrt{(x+1)(x-1)}$.

$$\therefore\ \sqrt{2x\pm 2\sqrt{x^2-1}}=\sqrt{x+1\pm 2\sqrt{(x+1)(x-1)}+x-1}=\sqrt{(\sqrt{x+1}\pm\sqrt{x-1})^2}$$
$$=\sqrt{x+1}\pm\sqrt{x-1}.$$ B.

10. If R be the radius of a sphere, prove that the edges of the five regular solids circumscribing and inscribing it are respectively as follows; namely, for the tetraedron, $2R\sqrt{6}$ and $\frac{2}{3}R\sqrt{6}$; for the

hexaedron, $2R$ and $\frac{2}{3}R\sqrt{3}$; for the octaedron, $R\sqrt{6}$ and $R\sqrt{2}$; for the dodecaedron, $R\sqrt{50-22\sqrt{5}}$ and $\frac{1}{3}R(\sqrt{15}-\sqrt{3})$; for the icosaedron, $R\sqrt{3}(3-\sqrt{5})$ and $\frac{1}{5}R\sqrt{10(5-\sqrt{5})}$. JOSEPH FICKLIN, Jr., Trenton, Missouri.

11. "If the digits of a given number be transposed in any order, the difference between the number thus formed and the given one is divisible by 9."

Any digit x of the given number will have some positive integer power, m, of 10, including zero, for its coefficient; and in the new number, if x has changed its place, it will have some other power, n, of 10 for its coefficient. Therefore, in the difference, the coefficient of x will be $10^m - 10^n$; and since the coefficients of all the other digits will have the same form in the difference, it will be sufficient to prove that $10^m - 10^n$ is divisible by 9. But $10^m - 10^n = (9+1)^m - (9+1)^n$, and, by expanding these binominals, the last terms of these expansions being unity with opposite signs, will cancel each other, while all the other terms will contain 9 as a factor; therefore $10^m - 10^n$ is divisible by 9, and the theorem is proved.

COR. By the same reasoning it appears that in a system of numeration of which the base is b, the highest digit, $(b-1)$, would be invested with the same property as 9 in our system. JOHN F. LANNEAU, Furman University, Greenville, South Carolina.

12. *Note on Logarithms.* A correspondent invites attention to the convenience of distinguishing between the direct and inverse use of logarithmic tables. When a number is given to find its logarithm, the sign of equality, or a blank space is commonly employed; thus, $l\,436.2 = 2.63969$; $l\,436.2\ \ 2.63969$; $l\,436.2 \ldots . 2.63969$. In the last case, the three or four dots imply that the number was found *from the logarithm.* When such a number or its logarithm is to be again employed in a case where greater accuracy is required, the

dots will show when the last decimals are to be reëxamined and corrected before making such application.

Since the prefix *log* is employed in analysis to denote Napierian or hyperbolic logarithms, the next most convenient designation which remains, is the italic *l* for designating common logarithms exclusively.

NOTE ON DIFFERENTIATION.

By Simon Newcomb, Naut. Alm. Office, Cambridge, Mass.

The following is an example of a method of finding a derivative by means of some special property of the function.

To find the derivative of a function which satisfies the condition

$$\psi(x+y) = \psi x \,.\, \psi y.$$

Representing the successive derivatives by ψ', ψ'', &c., we have

$$\psi'(x+y) = \psi x \,.\, \psi' y = \psi' x \,.\, \psi y.$$

That this equation may be satisfied, it is necessary and sufficient that we have

$$\psi' x = m \psi x,$$

m being a constant to be determined. By successive differentiation and substitution

$$\psi'' x = m \psi' x = m^2 \psi x$$
$$\psi''' x = m \psi'' x = m^3 \psi x$$
$$\vdots \qquad \vdots \qquad \vdots \qquad \&c.$$

Whence, by Maclaurin's theorem

$$\psi x = \psi 0 \left(1 + mx + \frac{m^2 x^2}{2!} + \frac{m^3 x^3}{3!} + \&c.\right).$$

Put $x = \frac{1}{m}$, and we have

$$\psi \frac{1}{m} = 2.71828\ldots. \; (\psi 0) \text{ which determines } m.$$

By the above method the function a^x may easily be differentiated. We have $D_x a^x = m a^x$; where $a^{\frac{1}{m}} = 2.71828\ldots$ $\therefore (2.71\ldots)^m = a$ or m is $\log a$ in the system of which the base is $2.7182818\ldots$.

THE MOTIONS OF FLUIDS AND SOLIDS RELATIVE TO THE EARTH'S SURFACE.

[Continued from page 373.]

43. The pressure of the atmosphere may be obtained from the first of equations (9). The terms in the equation depending upon the motions of the atmosphere are insensible, and consequently may be omitted. The term $D_t^2 r$ depends upon the acceleration or retardation of the vertical motion of the atmosphere, and is of the same order in comparison with g as the rate of its acceleration or retardation in comparison with that of a descending or ascending free body, and hence in all ordinary motions of the atmosphere it is insensible. Restoring the value of ω in (§ 2), the largest of the remaining terms is $2 r \sin^2 \theta\, n\, D_t \varphi$, which is of the same order in comparison with $r n^2$, as the east or west motion of the atmosphere in comparison with the motion of the rotation of the earth on its axis. But $r n^2 = \frac{1}{289} g$ only, hence the term is entirely insensible. We may therefore put

$$\frac{D_N P}{P} = -\alpha g.$$

Using the common system of logarithms and putting M for its modulus, we get by integration

$$\log P' - \log P = M \alpha g N, \tag{54}$$

in which P' is the pressure at the earth's surface.

Hence

$$D_\theta \log P = D_\theta \log P' - M g N D_\theta \alpha \tag{55}$$

By means of this equation the second of equations (9) becomes, by putting αP for k, (§ 6), and omitting the very small term $2 r D_t r D_t \theta$,

(56) $D_{\theta'} \log P' - Mg N D_{\theta'} \alpha = M\alpha [r^2 \sin\theta \cos\theta (2n + D_t \varphi) D_t \varphi - r^2 D_t^2 \theta]$.

44. In the case of the atmosphere, there must be a term in equations (9) to represent the resistances to the motions; and this term in the second of these equations may be denoted by $(\varphi) D_t \theta$ in the second member. Putting

(57) $$W = (\varphi) D_t \theta + r D_t^2 \theta,$$

W will represent the force which overcomes the resistances to the motions of the atmosphere between the equator and the poles, and also its inertia.

Since $D_t \varphi$ is generally very small in comparison with $2n$, it may be omitted in equation (56), which then becomes, by means of equation (57),

(58) $D_{\theta'} \log P' - Mg N D_{\theta'} \alpha = M\alpha (2 r^2 n \sin\theta \cos\theta D_t \varphi - r W)$.

45. It will be shown, that $r W$ is very small in comparison with $2 r^2 n \sin\theta \cos\theta D_t \varphi$; and hence, if $D_t \varphi$ were known, the preceding equation, neglecting the term $r W$, would give approximately the pressure of the atmosphere at the earth's surface. But since we do not know the value of the term denoting the resistances in the last of equations (9), we cannot determine the value of $D_t \varphi$; therefore, since $D_{\theta'} \log P'$ can be determined from observations of the barometric pressure, we shall use the equation to determine $D_t \varphi$, from which we easily obtain the east or west motion of the atmosphere. Denoting the velocity of this motion per hour by v, we shall have

(59) $$v = 3600 r \sin\theta D_t \varphi.$$

46. The ratio of the density to the elastic force decreases $\frac{1}{491}$ for

every degree of Fahrenheit. But as a higher temperature is always accompanied by a greater amount of aqueous vapor, the density of which is less than that of the atmosphere, the rate of decrease has been found to be $\frac{1}{449}$ for every degree. Let

α' be the value of α at the equator, and

i the difference of temperature between the equator and the poles.

If we suppose the temperature to decrease from the equator to the poles as the square of the sine of the latitudes, we shall have

$$\alpha = \alpha' (1 + \tfrac{1}{449} i \cos^2 \vartheta).$$

Hence

$$D'_\vartheta \alpha = -\tfrac{2}{449} \alpha' i \sin \vartheta \cos \vartheta.$$

By means of the last three equations, equation (58), putting R for r and e for $\frac{1}{M\alpha' g}$, is reduced to

$$(60)\quad v = \frac{1800}{R n \cos\theta (1 + \frac{1}{449} i \cos^2\theta)} \left(e g D_{\theta'} \log P' + \tfrac{2}{449} i g \sin \vartheta N + W \right).$$

Since the variation of α with the altitude can produce no sensible effect in the results, α has been regarded as a function of the latitude only. We must, therefore, take the mean value of α' belonging to the atmosphere at the equator at all altitudes, which we will assume to be that belonging to the temperature of 32°.

47. By means of observations of P at different altitudes, equation (54) gives the value of $\frac{1}{M\alpha g}$, which, at the temperature of 32°, has been determined to be 60156 feet; which, consequently, is the value of e. The difference between the mean temperatures of the equator and the poles is about 60°; we shall, therefore, in the following applications, put $i = 60$.

48. The value of $D_{\theta'} P'$ in the preceding equation can be determined approximately for any latitude from the preceding tables of barometric pressure. Since the coördinates of pressure given there

have been deduced from observations made in different longitudes and at all seasons, they are somewhat irregular; but coördinates can be assumed with regular differences, and such that the interpolated values of the coördinates of pressure for the latitudes given in the tables will very nearly correspond with the pressures given there; and then, from these coördinates, the approximate value of $D_{\theta'} \log P'$ can be determined. In this manner the values of $D_{\theta'} P'$ in the following table have been determined, except the first, which has been assumed. The third column of the table contains the values of v at the earth's surface, neglecting the term W, which will be shown to have, in general, a very little effect. The fourth contains the coefficient of N, and the fifth the value of v at the height of 3 miles.

TABLE.

Latitude.	$D_{\theta'} \log P'$.	v, ($N=0$).	Coeff. of N.	v, ($N=3$ miles).
75 N.	—.0060	— 2.7 miles.	2.33	+ 4.3 miles.
65	.0000	0.0	3.87	11.6
55	+.0188	+ 9.9	5.34	25.9
45	+.0080	+ 4.5	6.71	24.6
30	.0000	0.0	8.49	25.5
15	—.0060	—10.0	9.70	19.1
15 S.	+.0060	—10.0	9.70	19.1
30	—.0147	+11.4	8.49	36.9
40	—.0372	+23.4	7.36	45.5
50	—.0295	+15.3	6.07	33.5
60	—.0133	+ 6.0	4.61	19.8

49. The term W, and its effect upon the value of v, cannot be determined, but they can be shown from observation to be, in general, very small; and, since W is positive, as may be seen from equation (75), when the motion is from the north towards the south, and negative when the contrary, except when the motion is retarded, and the term $r D_t^2 \theta$ arising from the inertia of the atmosphere is greater

than the resistances, its effect for the most part is to increase the value of v algebraically where the motion is towards the south, and decrease it where it is towards the north. In the regions of the trade winds about the parallels of 15°, the current at the surface of the earth is stronger than at any other parallel, and as the resistances at the surface must be much greater than in the upper regions, the term W must be greater there than in any other part of the atmosphere. If $v = 0$, equation (60) gives, since $N = 0$ at the surface, $W = e\,g\,D_{\theta'} \log P'$; and from the preceding table, when $v = -10$ miles, $W = 0$. Now, we know from observation that the velocity of the atmosphere westward at the parallels of 15°, cannot be much less than 10 miles per hour, and hence W is small in comparison with $e\,g\,D_{\theta'} \log P'$, which at that parallel is itself small; and hence the effect of W upon the value of v in the higher latitudes, where the value of $\cos\theta$ in the denominator is much greater, must be very small. Very near the equator the formula for the value of v, equation (60), fails practically, since, on account of the small value of $\cos\theta$ there, the effect of W may be very great.

50. If the motion of the atmosphere east in the higher latitudes and west near the equator, be that given in the preceding table, or by equation (60), it must cause the observed difference of barometric pressure in the different latitudes; and hence, from what we know of those motions of the atmosphere from observations, there can be no doubt that they are adequate to account for this observed difference of pressure.

51. It is evident, where the motions of the atmosphere are resisted by the earth's surface, that all the conditions cannot be satisfied by a motion at the surface from the poles towards the equator, and by a counter motion in the upper regions. For we have seen (§ 35), that the atmosphere at the surface of the earth must have an eastern motion in the middle latitudes; but it cannot have such a motion,

unless it also have a motion towards the poles, in order that the deflecting force (§ 18) arising from this motion may overcome the resistances to the eastern motion. But it is evident that there cannot be a complete reversal of the motions in the middle latitudes, but some portion of it must flow towards the poles in the upper regions, else the eastern motion there could not be greater than at the surface, which the conditions require. The motions, therefore, must be somewhat as represented in Fig. (5). The part of the atmosphere next the earth's surface in the middle latitudes having a motion towards the poles, extends to a considerable height, since it generally embraces the region of fair weather clouds, as may be seen by observation.

52. It is seen, from the results given in the preceding table, that the eastward motion of the atmosphere in the middle and higher latitudes must be greatest in the upper strata, and that in the region of the trade winds, where the motion is westward at the surface, it must be towards the east above. This is also evident from the general consideration, that the whole amount of deflecting force eastward arising from the motion of the atmosphere towards the poles is equal to the deflecting force westward arising from its motion back towards the equator, and that the deflecting force eastward is principally above where there is less resistance than near the surface. Hence at the top of Mauna Loa in the Sandwich Islands, and on the peak of Teneriffe, both of which places are near the tropical calm belt at the surface, a strong south-west wind prevails. Hence, also, "on the eruption of St. Vincent, in 1812, ashes were deposited at Barbadoes, sixty or seventy miles eastward, and also on the decks of vessels one hundred miles still further east, whilst the trade wind at the surface was blowing in its usual direction." The eastward motion of the atmosphere above, in the latitudes of the trade winds, is also confirmed by observations made on the directions of the

clouds at Colonia Tovar, Venezuela, latitude 10° 26′, as given in the Report of the Smithsonian Institution for 1857 (p. 254). While the motion of the lower clouds was in general from some point towards the east, the observed motion of nearly all the higher clouds was from some point towards the west.

53. From what precedes, the limit between the atmosphere which moves eastward in the middle latitudes and westward nearer the equator, which at the earth's surface is at the tropical calm belt, must be a plane inclining towards the equator above. And since, according to (§ 51), the atmosphere near the earth's surface cannot have an eastward motion, unless it also has a motion toward the poles; this plane near the earth's surface must nearly coincide with the one which separates the atmosphere moving towards the poles from that moving towards the equator, in the trade wind regions, and hence the latter must also incline above towards the equator. This explains the winds at the peak of Teneriffe, which at the top always blow from the south-west, while at the base they blow alternately from the south-west and north-east, changing with the seasons. As the tropical calm belt together with this dividing plane changes its position with the seasons, as will be explained, in the latter part of summer when this plane is farthest north, it still leaves the top of the peak north of it while the base is south of it; and hence the wind at the top always blows from the south-west, even when at the base it blows from the north-east. As this plane moves south in the fall, more of the peak gradually becomes north of it; and hence the south-west wind, which always prevails at the top, gradually descends lower on the sides of the peak until it reaches the base. Hence, when this plane reaches its most southern position, in the latter part of winter, the south-west wind prevails at both the base and the top.

54. It is seen, from the first of the results given in the last table,

that if the barometric pressure increases near the poles, as it seems to do, at least in the northern hemisphere, the atmosphere at the earth's surface must have a westward motion there; and as it cannot have this motion unless it also have a motion toward the equator, so that the deflecting force arising from this motion may overcome the resistance to the westward motion, the wind there must blow slightly from the north-east, as has been shown in (§ 37). This, according to Professor COFFIN'S chart of the winds, already alluded to, seems to accord with observation.

55. The depression of the atmosphere at the poles and at the equator, and the accumulation near the tropics, may be explained in a general manner by means of the principle in (§ 32), that when a body moves in any direction in the northern hemisphere, it is deflected to the right, and the contrary in the southern. The atmosphere towards the poles having an eastward motion, the deflecting force arising from it causes a pressure towards the equator, and the motion near the equator being westward, the pressure is towards the poles; and hence there must be a depression at the poles and at the equator, and an accumulation near the tropics. Since this deflecting force is as $\cos \theta$, it is small near the equator; and, consequently the depression there is small.

56. According to the preceding tables of barometric pressure, there is more atmosphere in the northern than in the southern hemisphere. Says Sir JAMES ROSS, "the cause of the atmosphere being so very much less in the southern than in the northern hemisphere remains to be determined." This is very satisfactorily accounted for by the preceding principle; for as there is much more land, with high mountain ranges, in the northern hemisphere, than in the southern, the resistances are greater, and consequently the eastward motions, upon which the deflecting force depends, is much less; and the consequence is, that the more rapid motions of the

southern hemisphere cause a greater depression there, and a greater part of the atmosphere to be thrown into the northern hemisphere.

This also accounts for the mean position of the equatorial calm belt being, in general, a little north of the equator. But in the Pacific Ocean, where there is nearly as much water north of the equator as south, its position nearly coincides with the equator.

For the same reason the tropical calm belt of the northern hemisphere is farther from the equator than that of the southern hemisphere; and, on account of the irregular distribution of the land and water of the two hemispheres in different longitudes, it does not coincide with any parallel of latitude. In the longitude of Asia, where there is all land in the northern hemisphere and the Indian Ocean in the southern, this belt, which is also the dividing line which separates the winds which blow east from those which blow west, is farther from the equator than at any other place, as shown by Professor COFFIN'S chart.

57. In winter, the difference of temperature between the equator and the poles, upon which the disturbance of the atmosphere depends, is much greater than in summer; this causes the eastward motion of the atmosphere in either hemisphere during its winter to be greater, while in the other hemisphere it is less. Hence a portion of the volume of the atmosphere in winter is thrown into the other hemisphere; but, although the volume or height of the atmosphere is then less, yet, being more dense, the barometric pressure remains nearly the same. The difference at Paris, and in the middle latitudes generally, between winter and summer, is only about $\frac{1}{10}$ of an inch.

On account of this alternate change with the seasons of the velocity of the eastward motion of the atmosphere in the two hemispheres, the equatorial and tropical calm belts change their positions

a little, moving north during our spring, and south in the fall. When the sun is near the tropics, the true law of the decrease of temperature from the equator to the poles varies from that which has been assumed, (§ 46), and is then different in the two hemispheres, which doubtless has some effect also upon the position of the calm belts.

[To be Continued.]

METHOD OF SOLVING NUMERICAL EQUATIONS.

BY DASCOM GREENE,
Professor of Mathematics in Rensselaer Polytechnic Institute, Troy, N. Y.

THE most direct method of resolving a numerical equation of any degree, is to transform it in such a way that its first member becomes a perfect power of the same degree; the solution is then reduced to extracting the root of its second member.

If the first member can be made a perfect nth power by the addition or subtraction of a *number*, the solution will be effected by simply extracting the nth root of the absolute term.

Thus, if the given equation be

$$x^3 - 3x^2 + 3x = 15,$$

subtract . . . . . $1 = 1;$

then $$(x-1)^3 = x^3 - 3x^2 + 3x - 1 = 14,$$

$\therefore$ $$x - 1 = \sqrt[3]{14}, \text{ and } x = 1 + \sqrt[3]{14}.$$

If, however, it should be necessary to add terms involving x, then the quantity whose nth root is required, will be a polynomial, with numerical coefficients, and of a degree inferior to the nth. Thus, if we have

$$x^3 + 7x^2 + 5x = 18,$$

add $$8 - x^2 + 7x = 8 - x^2 + 7x;$$

then $(x+2)^3 = x^3 + 6x^2 + 12x + 8 = 26 - x^2 + 7x,$

$$\therefore \quad x = -2 + \sqrt[3]{26 - x^2 + 7x}.$$

We proceed, as before, to extract the root of the absolute term, but take care, during the process, to correct each partial dividend for the value of x already found.

Example 1. Find a root of the equation

$$x^3 + 2x^2 + 3x = 13089030.$$

Add $x^2 + 1 = x^2 + 1;$

then $x^3 + 3x^2 + 3x + 1 = 13089031 + x^2,$

and $x + 1 = \sqrt[3]{13089031 + x^2}.$

Hence, in proceeding to extract the cube root of the absolute term, each successive dividend must be increased by the value of x^2 corresponding to that part of the root already found. We have the following operation*:

		13089031	236 $= x+1$	$\therefore x = 235.$	
If $x = 200$,	$x^2 =$	40000			
		13129031			
		8	$= 2^3$		
		5129031	3×2^2	$= 12..$	12..
$230^2 - 200^2 = 30 \times 430 =$		12900	$3 \times 2 \times 3$	$= 18.$	36.
		5141931	3^2	$= 9$	27
		4167	$= 3 \times$	1389	1587..
		974931		$3 \times 23 \times 6$	$= 414.$
$235^2 - 230^2$		$= 2325$		6^2	$= 36$
		977256			
		977256			$= 6 \times 162876$

* We employ here the method of extracting roots explained in the January number of the Monthly, page 123, and which is originally due to Prof. George C. Whitlock, of Coburg, Canada West.

EXAMPLE 2. Find a root of the equation

$$x^4 - 3x^2 + 75x = 10000.$$

We have $x = \sqrt[4]{10000 + 3x^2 - 75x}$.

$10000 \mid 9.886\ldots = x$

$3 \times 9^2 - 75 \times 9 = 432$

9568

$9^4 = 6561$ $\qquad 4 \times 9^3 = 2916\ldots$ $\mid 2916\ldots$

3007.0000 $\qquad 6 \times 9^2 \times 8 = 3888..$ $\mid 7776..$

$3(9.8^2 - 9^2) - 75 \times 0.8 = 14.88$ $\qquad 4 \times 9 \times 8^2 = 2304.$ $\mid 6912.$

2992.1200 $\qquad 8^3 = 512$ $\mid 2048$

2662.6816 $\qquad = 0.8 \times 3328.352$ $\mid 3764768\ldots$ $\mid 3764768\ldots$

329.43840000 $\qquad 6 \times 98^2 \times 8 = 460992..$ $\mid 921984..$

$3(9.88^2 - 9.8^2) - 75 \times 0.08 = -1.2768$ $\qquad 4 \times 98 \times 8^2 = 25088.$ $\mid 75264.$

328.16160000 $\qquad 8^3 = 512$ $\mid 2048$

304.88948736 $\qquad = 0.08 \times 3811.118592$ $\mid 3857721088$

23.272112640000 $\qquad 6 \times 988^2 \times 6 = 35141184$

$3(9.886^2 - 9.88^2) - 75 \times 0.006 = -0.094212$ $\qquad 4 \times 988 \times 6^2 = 142272$

23.177900640000 $\qquad 6^3 = 216$

23.167419776016 $\qquad = 0.006 \times 3861.236629336$

This method of solving equations is of universal application, but its facility depends entirely on the skill of the computer, in making the necessary transformations. It is due, so far as I am aware, to Mr. JACOB HAFF, of Plumb Brook, Michigan.

A SECOND BOOK IN GEOMETRY.

REASONING UPON FACTS.

BY THOMAS HILL.

Continued from page 285.]

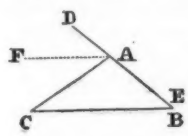
CHAPTER V.

VARIETY OF PATHS.

54. As there are usually many paths by which we may ascend a hill, so there are usually many modes by which we may demonstrate a proposition. In the case of a simple proposition, it is not usually worth while to try more than one mode. But with more difficult problems, it is sometimes worth while to spend a great deal of labor in discovering the simplest mode of demonstration. There are geometrical truths which can be demonstrated in so simple a manner as to require only twenty lines to write down the demonstration; and yet some writers, from ignorance of this simple mode, have written more than twenty pages to prove the same truths.

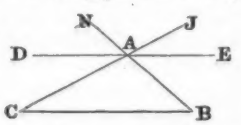
55. It will therefore be useful to you, to show you, by a simple example, such as that of the equality of the sum of the angles in a triangle to two right angles, the great variety of methods by which a single proposition can be proved.

56. In the proofs of this proposition, which I will now give you, I will not be careful to follow out every step. It will be enough, *for the purpose we now have in view*, simply to show you the general line of the paths, without taking you through every step of the way.



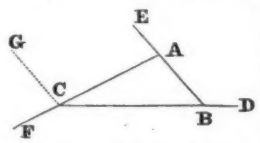
57. The line D E might be drawn so as to coincide in direction with one of the other sides of the triangle, as A B, which would give us the figure in the margin. And by imagining the dotted line A F parallel to C B, we should have F A C equal to A C B, and F A D equal to C B A, which would make the three angles at A equal to the three angles of the triangle, as in the former proof. (See chapter III.)

58. By prolonging the lines C A and B A through the point A, D E being parallel to B C, we should have N A D equal to C A B; J A E equal to A C B, and N A J equal to C A B. So that the three angles of the triangle will be equal to the three angles on the upper side of the line D E, which are manifestly equal to two right angles.



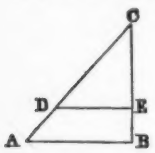
59. The three methods of proving this proposition that I have now given, are strictly geometrical. Others might be given, that are something more like algebraic reasoning.

60. Let us, for instance, imagine each side of a triangle prolonged at its right hand end, as in this figure, and also a line drawn from one vertex, as C, parallel to the opposite side B A. Now the external angles F C B, D B A, E A F, are plainly equal to the three angles F C B, B C G, G C F; and these amount to four right angles. But each external angle is plainly the supplement of one of the angles of the triangle; that is, it is equal to the difference between two right angles and one angle of the triangle. The sum of the three external angles must therefore be



equal to the difference between six right angles and the angles of the triangle. But as this difference is four right angles, the three angles of the triangle must be equivalent to two right angles.

61. If we introduce the idea of motion, we can devise quite a different sort of demonstration. Suppose, for instance, that I stand at the point A, with my face towards C. Let me now turn to the right until I face towards B. I have now changed the direction of my face by an amount which is equal to the angle at A. Suppose that I now walk to B, without turning; I shall have my back towards A, and if standing still, I turn to the right, until I have changed my direction by an amount equal to the angle A B C; I shall have my back towards C. Let me now walk backward without turning, until I reach C, and I shall have my face towards B. I will now turn a third time to the right, until I face the point A. My three turnings, or changes of direction, have been equal to the three angles of the triangle; they have all been to the right; therefore my whole change of direction is equal to the sum of these angles; I am now looking in exactly the opposite direction to that from which I started; I am looking from C to A, instead of from A to C; I have turned half way round; that is, through two right angles. Whence, the sum of the three angles of the triangle is equivalent to two right angles.



62. Another demonstration, by means of motion, may be obtained as follows. Suppose an arrow, longer than either side of the triangle, to be laid upon the side A C, pointing in the direction from A to C. Taking hold of the pointed end beyond C, turn the arrow round upon the point A, as a pivot, until the arrow lies upon the line A B. Taking now hold of the further end, beyond A, turn the arrow upon B as a pivot, until the arrow lies upon the line B C. Using C as a pivot, turn it now until the point of the arrow is over A. The arrow has thus been reversed in direction, turned half way round, or through two right angles. It has been turned successively through the three angles of a triangle, and every time in the same direction, like the hands of a watch; so that its total change of direction, two right angles, is equivalent to the sum of the three angles.

63. You have thus seen how a single proposition may be proved in a variety of ways. We have shown what is the value of the sum of the angles in a triangle, in six different ways; in three, by what is called rigid geometry, in one by a partly algebraical process; and in two, by introducing the idea of motion. And I wish you to observe, that every one of the six ways is satisfactory. They are all proofs that are certain; because they lead you from self-evident truths, by self-evident steps. One is not more certain than the other, because they are all absolutely certain. The only choice between them is, that some are more purely geometrical; some are better adapted to the peculiar tastes of different students, and some are neater and more quickly perceived by untaught persons.

Editorial Items.

THE following gentlemen have sent us solutions of the Prize Problems in the June number of the MONTHLY: —

JOHN W. JENKS, Senior Class, Columbia College, N. Y., answered questions III., IV., and V. (W. G. PECK, Prof.)

GEORGE A. OSBORNE, Jr., Lawrence Scientific School, answered all the questions. (H. L. EUSTIS, Prof.)

JAMES D. WARNER, West Troy, N. Y., answered all the questions but Problem IV.

ASHER B. EVANS, Madison University, Hamilton, N. Y., answered all the questions. (L. M. OSBORNE, Prof.)

D. D. ROSA, Georgetown, South Carolina, answered questions I. and II.

WILLIAM B. CLEVELAND, Lawrence Scientific School, answered all the questions. (H. L. EUSTIS, Prof.)

WE have, in one or two cases, unintentionally omitted to give credit for Solutions, which, we regret to say, have been mislaid. . . . . The Report of the Judges on the Prize Essays has been unavoidably delayed. It will be given in our next number. . . . . The Prize Problems are omitted in this number, in order to gain the month we have lost in the publication of Solutions. . . . . We wish to call the particular attention of our readers to the chapters entitled "Notes and Queries;" and earnestly beg of all, to aid us in making this one of the most interesting and valuable departments of the MONTHLY. Under this head we shall be able to make good use of a large amount of material already on hand, without consuming as much space as would be required for giving the same matter in short articles. . . . . In closing the first volume of the MONTHLY, we beg all, who have in any way aided us during the year, to accept our sincere thanks. Our duties, though not free from care, have yet been rendered pleasant by a considerate forbearance, not more for what has been done, than left undone. It is true that we have not been able to publish all the excellent matter we have received; but it is a satisfaction to know that the delay has not been misunderstood, and has caused us more anxiety than any of our contributors. The experiment of establishing a Mathematical Journal has been tried, and we point to Vol. I., as the most unmistakable evidence of success. While we acknowledge much valuable aid and counsel from friends in Cambridge and Boston, our special thanks are due to our friend and colleague, SIMON NEWCOMB, Esq., for the Indexes which accompany this number.

LIST OF ERRATA.

On page 19, lines —1 and —2, for ϱ_z, read ϱ_x.

" 19, " —3 and —4, for ϱ_z, read ϱ_y.

" 21, line 10, for $1+(D_x y)^2$, read $(1+(D_x y)^2)^{\frac{1}{2}}$.

" 229, " —5, in the value of y, put exponent $\frac{1}{2}$ outside the parenthesis.

" 231, " 11, for "A D." read "that line."

" 258, Prob. II., for "Transpose," read "Transform."

" 263, line —3, for "R," read "K."

" 282, " —12, for "Art. 1135," read "Art. 1535."

" 338, last line, for "> 0.250," read "< 0.250."

" 346, line 11, for "multipliers," read "multiples."

" " last line, for $(s-1)$, read $(s+1)$.

" 347, line 10, read, one entering only x, another only y, &c.; another only x and y, &c.; and one entering all $x, , z$.

" 348, Equation (11), for Q^{P_0} read $a^{\lambda'}$, and insert

$$\lambda' = M(F\overline{\xi,\eta},\ F\overline{\xi,\zeta},\ F\overline{\eta,\zeta}) = F(M\overline{\xi,\eta},\ M\overline{\xi,\zeta},\ M\overline{\eta,\zeta}).$$

" 350, for 63 cents, read 36 cents.

" 360, $2r = d = 2R - a\pi \pm \sqrt{a^2\pi^2 + 4al - 8a\pi R}$, instead of the value there erroneously printed.

" 361, in the last value given of d, the parenthesis should be multiplied by $2a$, instead of 2.

" 373, Equation (2), take $\frac{1}{2}$ the radical.

" 374, line —4, for $x' = x$, $y = 1$, read $x' = 1$.

" 375, " —7, for "the value of d," read "two of the six values of d."

" 377, " —6, for $+px$, read $-px$.

" 378, line —3, for $-px$ read, $+px$.

" 379, Equation (28), for $\frac{1}{q}$ read $\frac{1}{9}$.

" 379, line —5, for "dropped," read "developed."

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